Comparison of various numerical schemes for simulating fluid flow in variably saturated porous media

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Introduction and Objectives

- Prediction of fluid movement in unsaturated soil using Richards equation
- Nonlinear Richards equation in variably saturated porous media connected with constitutive relations, heterogeneities, irregular geometries, and complex boundary conditions
- Difficulty in solving the Richards equation originated from its nonlinear characteristic
- To demonstrate a full linearization of the Richards equation with Gardner constitutive relations by using Kirchhoff integral transformation in a transient variably saturated flow
- To show that the integral transformation approach is not only more computationally efficient but also more robustness than other existing numerical methods: h-based model, Celia model (Celia et al., 1990), Kirkland model (Kirkland et al., 1992), and 3DFEMWATER (Yeh et al., 1992)

Mathematical Formulation

Governing Equation

\[ \nabla \cdot (K \cdot \nabla (h + z)) + Q = \frac{\partial h}{\partial t} \]

Gardner Constitutive Relations

\[ \phi = K \cdot (\phi_{\text{sat}} - h)^{k} \]

Boundary Condition

\[ n \cdot \nabla \cdot (K \cdot \nabla (h + z)) = \frac{\partial h}{\partial t} \]

Kirchhoff Integral Transformation

\[ \phi = \frac{1}{h} \int_{0}^{h} \frac{\partial h}{\partial t} \, dh \]

Linearized Governing Equation and Boundary Conditions

\[ \nabla \cdot (K \cdot \nabla (h + z)) + Q = \frac{\partial h}{\partial t} \]

Numerical Implementation

\[
\begin{align*}
[CK]\frac{\partial}{\partial t} + [DKV] \cdot \nabla h + [G] = 0 \\
[CK] = \sum_{j=1}^{N} V_{j} \cdot K \cdot V_{j} \cdot \nabla \phi, \\
[DKV] = \sum_{j=1}^{N} V_{j} \cdot K \cdot \nabla V_{j} \\
[G] = \sum_{j=1}^{N} V_{j} \cdot \nabla \phi \cdot \nabla h + \sum_{j=1}^{N} V_{j} \cdot K \cdot \nabla \phi \cdot \nabla h
\end{align*}
\]

Numerical Experiments

Example 1

This example is selected to represent the simulation of a one-dimensional homogeneous problem. The conceptualization of the problem is given in terms of initial and boundary conditions and material properties in Fig. 1. For numerical simulations, the h-based model and Kirchhoff integral transformation method are given. For this problem, water is applied to the top of a vertical soil column at a constant rate of 20 cm/day for about 70 min. The unsaturated characteristic hydraulic properties of the soil in the column are characterized with Gardner relationship.

Example 2

Fig. 2 illustrates very good agreement between the two methods. CPU with Kirchhoff transformation is faster 1.7 times than with h-based model.

Example 3

A basic case solution to a Richards equation was calculated for the purpose of comparing and evaluating the six models.

Conclusion

- Kirchhoff integral transformation in a transient variably saturated flow makes Richards equation with Gardner constitutive relations full linearized form
- Kirchhoff integral transformation approach is not only more computationally efficient but also more robustness than other existing models

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